Analysis and Design of an Adaptive Parameter Estimator for Power Electronics Circuits

Jason Poon and Seth R. Sanders
Department of Electrical Engineering and Computer Sciences
University of California, Berkeley
Berkeley, CA 94720
Email: jason@berkeley.edu, seth.sanders@berkeley.edu

Abstract—This paper presents the analysis and design of an adaptive parameter estimator for power electronics circuits. Adaptive parameter estimation has been demonstrated to be a useful technique for enabling novel controls, monitoring, and fault diagnosis techniques in power electronics systems. We present an analysis of factors that affect the performance, stability, and feasibility of a gradient-type parameter estimation, along with simulation and experimental results that validate the analysis. Moreover, we derive theoretical limits on the convergence speed of the parameter estimation and demonstrate design techniques to maximize this limit while maintaining the robustness of the estimation. The paper provides a comprehensive analysis and design process for adaptive estimation schemes as applied to systems of switching power converters.

I. INTRODUCTION

Adaptive parameter estimation is a technique in which system parameters, such as passive component values, are estimated online, typically employing a gradient-type algorithm to establish exponential convergence of a parameter estimate error vector to zero. Adaptive estimation has been demonstrated to be a useful tool for power electronics systems, providing methods for estimation-based control strategies [1], fault prognosis and component wear detection [2], and condition monitoring for photovoltaic systems [3], among others.

In order for adaptive estimation schemes to be applied to power electronics circuits or systems, one must consider a number of factors that affect the performance, stability, and feasibility of the parameter estimation. These factors include: 1) persistence of excitation (P.E.) of the regression variable, 2) limits on speed of the convergence of the parameter estimation error vector to zero, 3) selection of the adaption gain, 4) selection of measurement outputs, and 5) optimal input design. The majority of these topics have not been discussed in the context of power electronics circuits and systems in existing literature. Traditionally, considering these factors would vastly increase the design complexity of the adaptive estimator for a switching power converter, and would make the approach impractical for many applications.

The objective of this paper is to provide a comprehensive analysis and straightforward design procedure for adaptive estimation schemes as applied to power electronics systems. We derive theoretical limits on the speed of convergence of the parameter estimation, and propose techniques to design the estimator and the inputs to the estimator as to maximize the speed of convergence while being immune to measurement noise and other non-idealities.

The procedure requires only a numerical simulation of the power electronics system under test, as well as a set of equations that are derived in this paper. The proposed design procedure obviates the complexity of considering the individual factors that influence performance, stability, and feasibility of the parameter estimation.

The remainder of the paper is outlined as follows. Section II presents an overview of the gradient-type parameter estimator for a power electronics system. Section III presents a derivation of a guaranteed lower limit on the rate of convergence of the parameter estimate, along with simulation and experimental results that demonstrate the validity and usefulness of the derived limit. Section IV discusses some of the factors that can influence and potentially maximize this lower limit (that is, speed up the convergence), including the dynamics of the input vector, the switching frequency of the power converter, and the selection of the output measurement vector. Section V presents a design process for selecting the adaption gain, and discusses the resulting trade-offs between robustness and convergence speed. Section VI concludes the paper.
II. OVERVIEW OF ADAPTIVE PARAMETER ESTIMATOR ALGORITHM

This section presents an overview of the design of a gradient-type adaptive parameter estimator for an arbitrary power electronics system. The adaptive estimator generates an estimate of parameters of interest in the system (e.g., the values of passive components, equivalent series resistances, etc.), and dynamically updates this estimate by comparing output measurements from the converter with an internal state estimator model. The objective of the adaptive estimator is to perturb the estimate of the parameters as to drive the error between the output measurements and the internal state estimator model to zero, and by doing so, obtain an accurate estimate of the parameters of interest.

The adaptive estimator is comprised of three main components:

1) A regression variable,
2) A switched linear state estimator, and
3) A parameter update scheme.

Each of these components is developed in detail in the subsequent subsections.

A. Regression variable

First, one identifies the parameters of interest within the system. These could be parameters within the converter itself (e.g., capacitances, inductances) or parameters in systems surrounding the converter (e.g., network impedances, electrochemical constants). These parameters are assembled into a time-varying vector of estimates denoted \( \theta(t) \). The actual ‘true’ values of these parameters is denoted as \( \theta^* \).

The state dynamics of the converter are parametrized with respect to \( \theta(t) \). In general, the state dynamics of an ideal switching power converter can be represented as a switched linear state space system, that is:

\[
\dot{x}(t) = A_\sigma(t)x(t) + B_\sigma(t)u(t) \tag{1}
\]

\[
y(t) = Cx(t) \tag{2}
\]

where \( x(t) \) and \( y(t) \) are the states and outputs of the system respectively, \( u(t) \) are the inputs to the system, \( \sigma(t) \) is a switching signal that indicates the active mode of the switched system based on the on/off configuration of active and passive switches in the power converter, and \( A_\sigma(t), B_\sigma(t), \) and \( C \) are the associated matrices that describe the true state dynamics of the system with parameters from \( \theta^* \). The structure of \( y(t) \) and \( C \) depend on the available sensor measurements and their associated bandwidth.

This switched linear state space model can be parameterized with respect to the vector \( \theta^* \) as follows:

\[
A_\sigma(t)x(t) + B_\sigma(t)u(t) = W_\sigma(t)(x, u) \theta^* \tag{3}
\]

where \( W_\sigma(t)(x, u) \) is a matrix that varies as a function of \( x(t) \) and \( u(t) \).

Next, we define the regression variable \( H(t) \). As will be shown in Sections III and IV, the regression variable is an important variable that yields information about the feasibility and rate of convergence of the adaptive estimator. The dynamics of the regression variable are defined as follows:

\[
\frac{d}{dt}H(t) = A_\sigma(t)H(t) + W_\sigma(t)(x, u) \tag{4}
\]

B.Switched linear state estimator

We construct an internal state estimator that generates an estimate of the states of the system \( x(t) \) using the parameter estimates in \( \theta(t) \). We compare the outputs of this state estimator with the measured outputs \( y(t) \) to form an output error vector \( \gamma(t) \).

Let \( \hat{A}_\sigma(t) \) and \( \hat{B}_\sigma(t) \) be the matrices \( A_\sigma(t) \) and \( B_\sigma(t) \) but with parameters from \( \theta(t) \) instead of \( \theta^* \). The internal state estimator is defined as follows:

\[
\dot{z}(t) = \hat{A}_\sigma(t)z(t) + \hat{B}_\sigma(t)u(t) \tag{5}
\]

\[
\gamma(t) = Cz(t) - y(t) \tag{6}
\]

where \( z(t) \) is the vector of state estimates.

C. Parameter update scheme

Lastly, we will design an update scheme that causes the parameter estimate vector \( \theta(t) \) to exponentially converge to the true parameter values \( \theta^* \).

Let \( \phi(t) \triangleq \theta(t) - \theta^* \) be the difference between the estimate \( \theta(t) \) and the actual \( \theta^* \) values of the unknown parameters. Intuitively, we see that if \( \phi(t) \equiv 0 \), then \( \hat{A}_\sigma(t) \equiv A_\sigma(t) \) and \( \hat{B}_\sigma(t) \equiv B_\sigma(t) \). This indicates that the dynamics of (5) and (1) are identical, which implies that the output error vector in (6) will be zero (\( \gamma(t) \equiv 0 \)). Conversely, if \( \gamma(t) \) is large, then \( \phi(t) \) would also be large.

The gradient descent algorithm [1], [4] is a popular method for parameter estimation problems due to its simplicity and guarantees of exponential convergence if certain conditions are satisfied. The implementation of the gradient algorithm is as follows:

\[
\dot{\theta}(t) = -G H^T(t) \gamma(t) \tag{7}
\]

where \( G \succ 0 \) is a positive definite diagonal matrix called the adaption gain. Further analysis of (7) including
limits and guarantees of convergence are presented in Section III.

D. Complete adaptive identifier structure

In its complete form, the identifier structure is expressed as:

\[ \dot{z}(t) = \hat{A}_\sigma(t)z(t) + \hat{B}_\sigma(t)u(t) \]  
(8)

\[ \gamma(t) = C z(t) - y(t) \]  
(9)

\[ \frac{d}{dt} \hat{H}(t) = \hat{A}_\sigma(t) \hat{H}(t) + W_\sigma(t)(z, u) \]  
(10)

\[ \dot{\theta}(t) = -G \hat{H}^T(t) \gamma(t) \]  
(11)

Note that (10) uses the estimates \( \hat{A}_\sigma(t) \) and \( \dot{z}(t) \) as these are the values that are available to the adaptive identifier.

In order to validate the adaptive identifier, we performed a numerical simulation using one phase of the interleaved boost converter shown in Fig. 1. The parameters for the converter are as follows: \( R_1 = 0.82 \Omega \), \( L_1 = 5 \text{ mH} \), \( C = 100 \mu \text{F} \), \( v_1 = 100 \text{ V} \), \( \epsilon_C = 5 \cdot 10^4 \), and \( f_{sw} = 50 \text{ kHz} \). The load \( i_{load}(t) \) is modeled as a 5 A DC current with a superimposed 1 A pk-pk 16.7 Hz ripple. The estimated state variables are \( z(t) = [i_{L1}(t) \ v_C(t)]^T \) and \( \mathbf{C} = [1 \ 0] \).

The results of the simulation are shown in Figs. 2a. At \( t = 0.04 \text{ s} \), the value of the output capacitor \( C^* \) is decreased from 100 \( \mu \text{F} \) to 65 \( \mu \text{F} \). As shown, the estimated parameter \( C(t) \) converges to the new value of \( C^* \) in approximately 0.15 s.

Similarly, the results of the experimental implementation are shown in Fig. 2b. The implementation details of the experiment are identical to the values used in the simulation. Details on the experimental setup can be found in [2]. At \( t = 0.2 \text{ s} \), the value of the output capacitor \( C \) is decreased using a step function by removing one of two parallel capacitors from the circuit. As shown, \( C(t) \) converges to \( C^* \) in approximately 0.6 s. Although the circuit and parameter identifier are identical to the simulation, the convergence time is approximately three times as long. Indeed, the practical implementation details affect the convergence rate as will be shown in subsequent sections. Particularly, the finite bandwidth of the voltage and current sensors as well as the finite quantization resolution of analog-to-digital converters plays an important role in distinguishing the ideal simulation from a practical realization of the parameter estimator.

E. Optimization and design procedure

The proposed optimization and design procedure for the adaptive parameter identifier is shown in Fig. 3. The procedure has two objectives: (1) maximize the convergence speed of the parameter identifier, and (2) maximize the robustness to noise and non-idealities. These objectives require two separate techniques for...
determining if the parameter identifier will meet the required specification for both convergence speed and robustness. The technique for determining the feasibility and speed of convergence are presented in Section III and IV. The technique for designing the adaptive gain to ensure robustness is presented in Section V. Both techniques have low computational complexity and can be computed in real-time to continually tune the parameter identifier, as well as provide confidence intervals on the value of the parameter estimates.

III. LIMITS ON CONVERGENCE

In this section, we present an analytical method for: 1) validating the feasibility of convergence of the adaptive parameter identifier, and 2) predicting the speed of the convergence of $\phi(t)$ to zero, that is a lower limit of the rate of convergence.

These metrics can be used to calculate an online confidence interval over a sampling window $T$ (typically 5 to 10 ms) that indicates whether the estimation problem is tractable at a given operating point and if the parameter estimates are valid. The size of this sampling window is chosen to be at least an order of magnitude larger than the time scale of the power electronics circuit. The lower limit on the rate of convergence can inform when a given estimate would be valid in response to dynamics in the system (e.g. the sudden change of a parameter value in a system).

A necessary condition for the convergence of $\phi(t)$ is:

$$
\lambda \left\{ \int_{t}^{t+T} \hat{H}^T(\tau)\hat{H}(\tau)d\tau \right\} > 0 \ \forall \ t \geq t_0
$$

that is, the eigenvalues of $\int_{t}^{t+T} \hat{H}^T(\tau)\hat{H}(\tau)d\tau$ must be positive and bounded for all $t \geq t_0$ over some $T$. This encapsulates the persistence of excitation (P.E.) condition and guarantees the positive definiteness of $\hat{H}^T\hat{H}$ [4].

Practically, this condition implies that over some sampling window $T$, the transients in the states $x(t)$ and the input $u(t)$ (e.g. load or source transients) must be sufficiently changing in order to enable the tractability of the parameter identification problem. For instance, a converter that is not switching and does not have time-varying input dynamics would not yield an identifiable
problem. Typically, this condition can be satisfied by the switching dynamics of the converter through the switching signal \( \sigma(t) \). However, as will be shown in Section IV, the dynamics of the input \( u \) have a larger impact on satisfying the convergence condition.

Next, we can determine the lower limit on the rate of convergence of the parameter identification. Let \( \theta_i(t) \) be the \( i^{th} \) parameter estimate in \( \Theta(t) \). We can determine that the slowest rate of convergence \( k_i \) of \( \theta_i(t) \) is given by:

\[
k_i = \epsilon_i \lambda_{\min} \left\{ \lim_{T \to \infty} \frac{1}{T} \int_0^T \mathbf{H}^T(\tau) \mathbf{H}(\tau) d\tau \right\}
\]

(13)

where \( \epsilon_i \) is the \( i^{th} \) diagonal element in \( \mathbf{G} \). Thus, the lower limit on the rate of convergence is:

\[
|\theta_i(t + \tau)| \leq K_i e^{-k_i \tau} |\theta_i(t)| \forall t, \tau
\]

(14)

for some positive \( K_i \) [5].

In order to validate the lower limit determined by Eq. 13, we calculated \( k_i \) for the simulation and experimental condition in Fig. 2. As shown in Fig. 4a, the lower limit on the rate of convergence of \( \phi_C(t) \) to zero calculated from Eq. (13) provides an accurate limit of the simulated \( \phi_C(t) \). Similarly, the results of the experimental implementation are shown in Fig. 4b. The calculated lower limit from Eq. (13) naturally accounts for the factors that cause the rate of convergence for the experimental implementation to be slower than that of the ideal simulation, and provides a close estimate of the convergence time of \( \phi_C(t) \).

Together, these metrics from (12) and (13) can be repeatedly computed over sampling windows of size \( T \) with low computational overhead to provide a confidence interval on the accuracy and predicted convergence times of the estimates \( \Theta(t) \).

IV. EFFECTS ON CONVERGENCE LIMIT

The convergence speed of the adaptive parameter estimation can be highly dependent on the operating condition of the power electronics system. In this section, we analyze factors that influence the rate of convergence of \( \phi(t) \) to zero. Our objective is to determine scenarios that will maximize the lower limit on the rate of convergence \( k_i \) in Eq. (13), which will in effect speed up the parameter estimation.

While analytic solutions to maximize the convergence speed have been attempted for the more general adaptive parameter identification problem [5], the methods typically lead to an optimal input design problem, which are not practical or tractable for a power electronics system. Thus, we use the limit derivation presented in Section III to determine the impacts of three different operating factors most relevant to the operating condition of a power electronics converter. Once these impacts are quantified, we can choose optimal operating conditions for the power electronics system that will increase the convergence speed of the parameter identification.

The operating factors we consider are: 1) the effect of output measurement selection \( y(t) \), 2) the effect of switching dynamics \( \sigma(t) \), and 3) the effect of input dynamics \( u(t) \).

As a benchmark, we calculate the time \( \tau_c \) required for the value of \( \theta_i(t) \) to reach within ten percent of the actual value of \( \theta^* \) for a step change in the \( C^* \) parameter value. This can be simply calculated as \( \tau_c = \frac{\log(0.1)}{-k_i} \) at the instance the step parameter change is applied. The simulation framework used is the same as in Section II.

A. Effect of output measurement selection \( y(t) \)

First, we explore the effect of the available sense variables in the output measurement vector \( y(t) \). As shown in Fig. 5, by sensing only one inductor current \( i_L(t) \), convergence of \( \phi_C(t) \) is possible, but is approximately two orders of magnitude slower than by sensing the capacitor voltage \( v_C(t) \).

The selection of \( y(t) \) directly affects the regression variable \( \mathbf{H}^T \) through the output error \( \gamma(t) \). However, the result of this effect may not be clear from inspection without relying on more complicated tools from observability theory. The analysis of the lower limit \( k_i \) yields a simple and definitive method for predicting the effect of sensor selection on \( \tau_c \).

B. Effect of switching dynamics \( \sigma(t) \)

The switching dynamics of controlled switches (e.g. MOSFETs, IGBTs) and uncontrolled switches (e.g. diodes) manifest through \( \sigma(t) \). To explore the impact on
Fig. 6: Convergence time as a function of the switching frequency of the dc-dc converter.

Fig. 7: Convergence time as a function of the ripple ratio of \( i_{load}(t) \).

As shown, the ripple ratio has a drastic effect on the rate of convergence. With no ripple in \( i_{load}(t) \), \( \tau_c = 1.483 \) s. With a ripple ratio of 5 in \( i_{load}(t) \), \( \tau_c = 0.0409 \) s. Since \( u(t) \) manifests in \( \mathbf{W}_{\sigma(t)}(z, u) \), it has a direct impact on the regression variable \( \hat{\mathbf{H}}(t) \) and thus strongly influences the rate of convergence of \( \phi(t) \).

In this way, one can expect that the parameter identifier will have a faster convergence when there are more transients in the inputs and outputs of the system, as this will increase the persistency of excitation of the regression variable. Moreover, an analysis of the input power spectrum of \( u(t) \) can provide useful insight into the convergence speed of \( \phi(t) \).

V. ADAPTATION GAIN DESIGN

The adaption gain is a design variable that balances the trade-off between convergence speed of the parameter identification and its robustness to noise and non-idealities.

The adaption gain has a linear relationship with the convergence speed, since it appears as a scalar term in the convergence limit in (13). As shown in Fig. 8, the effect of increasing \( G \) has a natural consequence of increasing the rate of convergence. Thus, it is desirable to have a high gain in order to minimize the time to convergence of the parameter estimate.

However, as \( G \) increases, the adaptive estimation becomes more susceptible to measurement noise and model non-idealities [5]. The amount of this susceptibility is proportional to \( \beta G \), where:

\[
\beta = \lambda_{max} \left\{ \lim_{T \to \infty} \frac{1}{T} \int_0^T \mathbf{H}^T(\tau)\mathbf{H}(\tau) d\tau \right\} 
\]

In general, the initial choice of the adaption gain should be chosen such that \( \beta G \) is small enough to satisfy the assumption that the parameter variables are changing slowly with respect to the state variables. Then, an iterative approach can be used to determine the best-case...
performance scenario of the adaptive estimator given the specifications and operating conditions of the system.

VI. Conclusion

This paper presented an analysis and design procedure for a gradient-type adaptive parameter estimator for power electronics circuits. We presented theoretical limits on the performance and stability of the parameter estimation, along with simulation and experimental results that validate the analysis. Methods for improving the convergence speed and robustness of the parameter estimator are also discussed. We propose design guidelines for the adaption gain and regression variable to balance trade-offs between performance and robustness of the parameter estimation.

References


